

Integral of $x^4 \cos x$

This problem provides a lot of practice with integration by parts.
Compute the integral of $x^4 \cos x$.

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$$\begin{aligned}
 & \int x^4 \cos x \, dx \quad \begin{array}{l} u = x^4 \\ v' = \cos x \end{array} \\
 &= x^4 \sin x - \int 4x^3 \sin x \, dx \\
 &= x^4 \sin x + 4x^3 \cos x - \int 12x^2 \cos x \, dx \\
 &= x^4 \sin x + 4x^3 \cos x - 12x^2 \sin x \\
 &\quad + \int 24x \sin x \, dx \\
 &= x^4 \sin x + 4x^3 \cos x - 12x^2 \sin x \\
 &\quad - 24x \cos x + \int 24 \cos x \, dx
 \end{aligned}$$

$$\begin{aligned}
 & \int 4x^3 \sin x \, dx \\
 &= 4x^3(-\cos x) - \int 12x^2(-\cos x) \, dx \\
 & \int 12x^2 \cos x \, dx \\
 &= 12x^2 \sin x - \int 24x \sin x \, dx \\
 & \int 24x \sin x \, dx \\
 &= 24x(-\cos x) - \int 24(-\cos x) \, dx \\
 & \int 24 \cos x \, dx \\
 &= 24 \sin x + C
 \end{aligned}$$

$$\therefore \int x^4 \cos x \, dx = x^4 \sin x + 4x^3 \cos x - 12x^2 \sin x - 24x \cos x + 24 \sin x + C$$